Structural Analysis: Intermediate Shaft

Power Transmission Case Study

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Structural Analysis Objective and Method

- Objective: Perform a first-order static stress analysis of the intermediate shaft component to assess structural integrity.
- Method: Finite element analysis using SolidWorks Simulation.

Finite Element Analysis – steps[1]:

1. Preprocessing:
    - Geometry
    - Material properties
    - Loads
    - Boundary conditions
    - Element type
    - Mesh density
2. Numerical analysis:
    - Solve stiffness matrix (software solves automatically to determine values of field quantities at nodes).
3. Postprocessing:
    - Stress plots

[1]See Cook, R.D., *Concepts and Applications of Finite Element Analysis,* 4th ed., Wiley, Hoboken, NJ, 2002, p. 13.

1. Preprocessing

- **Geometry**
  - Stepped shaft

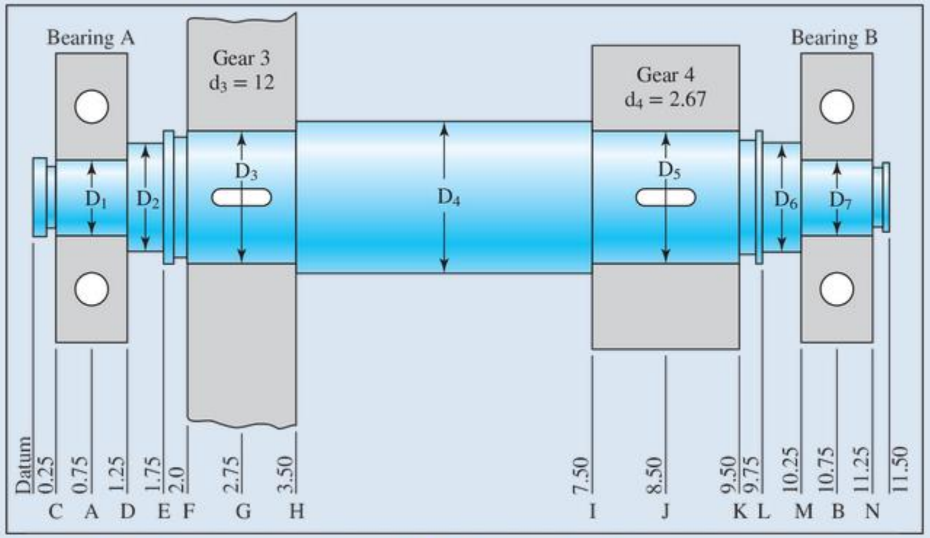
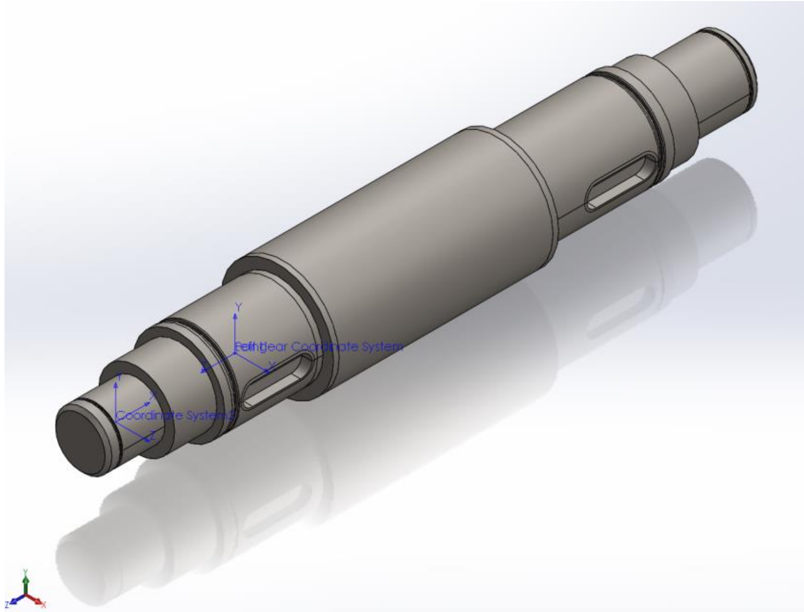
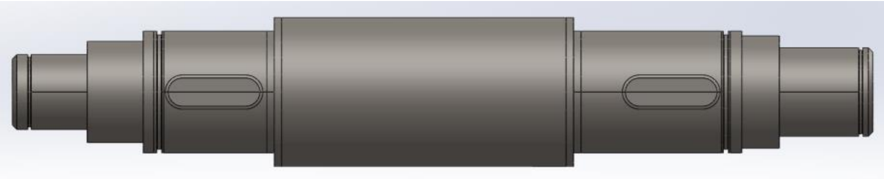


Figure 7–10 Shaft layout for **Example 7–2**. Dimensions in inches.



Isometric View



Front View

[1]See Budynas, R.G., *Shigley's Mechanical Engineering Design,* 8th ed., McGraw-Hill, New York, NY, 2008, pp. 362.

1. Preprocessing, cont.

- **Material (Properties) Selection**
  - AISI (American Iron and Steel Institute) 1045 Steel, cold drawn[1]
    - Note: Deflection is not affected by strength, but rather by stiffness (modulus of elasticity), which is essentially constant for all steels.
    - Shafts should be surface hardened if they serve as the journal of a bearing surface.
    - Cold drawn steel is usually used for diameters under 3 inches.
  - Applications: Where high strength and wear resistance are required
    - E.g. Gears, shafts, axles, spindles, pins, guide rods, connecting rods, bolts, machine components, etc.
  - *Note that the following simulation is performed under the assumption of isotropic and homogenous material properties.*

Material Properties:

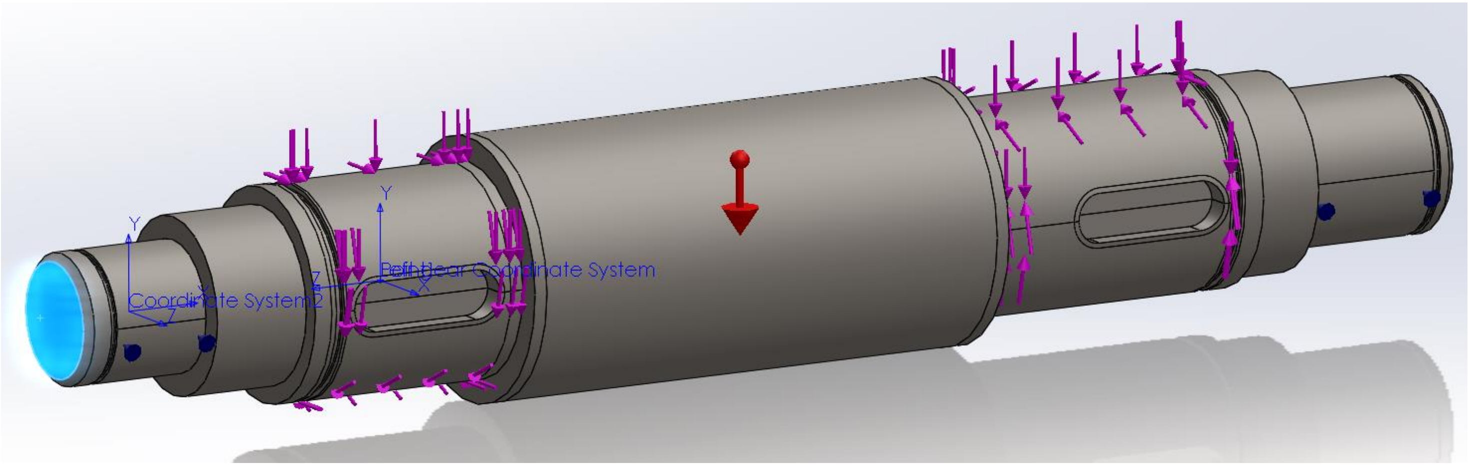
Property	Value	Units
Elastic Modulus	29732736.22	psi
Poisson's Ratio	0.29	N/A
Shear Modulus	11603019.01	psi
Mass Density	0.2835991622	lb/in^3
Tensile Strength	90648.58604	psi
Compressive Strength		psi
Yield Strength	76870.00096	psi
Thermal Expansion Coefficient	6.388888889e-06	/ºF

ASTM SAE AISI 1045 steel

[1]See Budynas, R.G., *Shigley's Mechanical Engineering Design,* 8th ed., McGraw-Hill, New York, NY, 2008, pp. 348 – 349.

1. Preprocessing, cont.

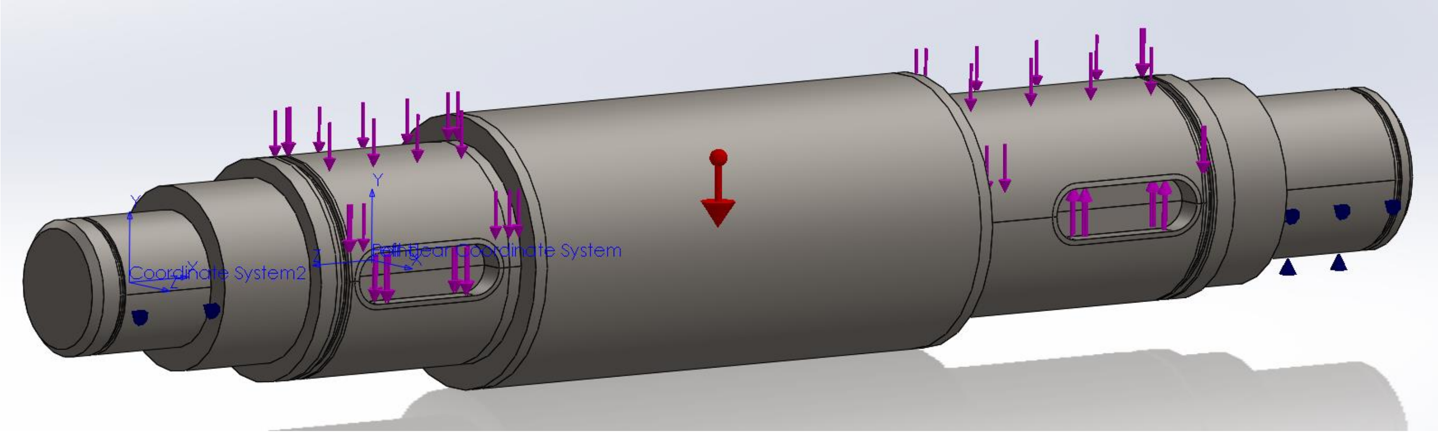
- **Loads**
  - Load case 1:
    - Bearing load 1; radial load exerted by gear 2 on 3 = 197 lbf
      - Applied normal to half of the gear 3 shaft along the negative y-axis
    - Bearing load 2; radial load exerted by gear 5 on 4 = 885 lbf
      - Applied normal to half of the gear 4 shaft along the negative y-axis
    - Torque = 3,240 lbf-in
      - Applied to gear 3 shaft, clock-wise as seen from left-end (torque input)
      - Applied to gear 4 shaft, counter-clock-wise as seen from left-end (torque output)
    - Gravitational acceleration = 386.22 in/s^2



Load case 1: Radial loads and torque

1. Preprocessing, cont.:

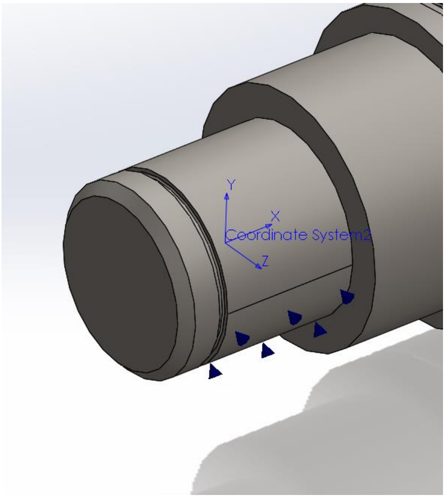
- **Loads**
  - Load case 2:
    - Bearing load 1; radial load exerted by gear 2 on 3 = 197 lbf
      - Applied normal to half of the gear 3 shaft along the negative y-axis
    - Bearing load 2; radial load exerted by gear 5 on 4 = 885 lbf
      - Applied normal to half of the gear 4 shaft along the negative y-axis
    - Force 1; transmitted load exerted by gear 2 on 3 = 540 lbf
      - Applied normal to keyway side-wall along the negative y-axis
    - Force 2; transmitted load exerted by gear 5 on 4 = 2,431 lbf
      - Applied normal to keyway side-wall along the positive y-axis
    - Gravitational acceleration = 386.22 in/s^2



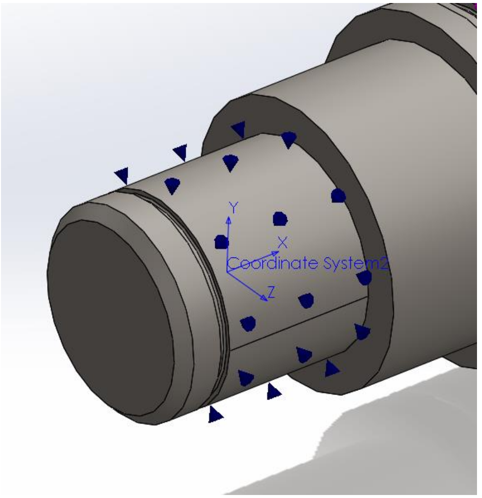
Load case 2: Radial loads and transmitted loads

1. Preprocessing, cont.

- **Boundary Conditions**
  - Two types of boundary conditions are applied at the bearing shafts: simply supported and fixed
  - A shaft with bearings is more likely to have boundary conditions that exhibit behavior *between* simply supported and fixed[1].
  - Therefore, the results are bounded by two analyses[2].
    - One with simple supports, which will **overestimate** the magnitude of the actual bending moment at midspan.
    - The second with fixed supports, which will **underestimate** the magnitude of the actual bending moment at midspan.



Simple supports

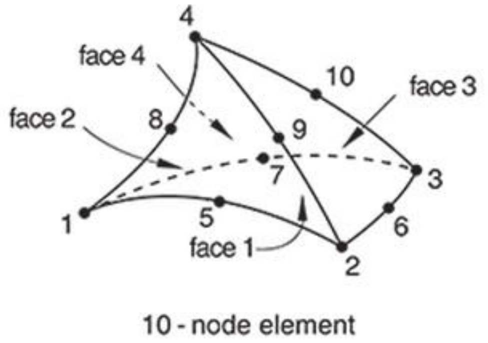


Fixed supports

[1]See Budynas, R.G., *Shigley's Mechanical Engineering Design,* 8th ed., McGraw-Hill, New York, NY, 2008, pp. 945 – 946.
[2] See Cook, R.D., *Concepts and Applications of Finite Element Analysis,* 4th ed., Wiley, Hoboken, NJ, 2002, p. 352.

1. Preprocessing, cont.

- **Element Type**
  - Tetrahedra
    - Number of nodes per element: 16
    - Note: Four-node tetrahedron are susceptible to shear locking behavior; higher-order elements – e.g. 16-node tetrahedron – are preferable choices for stress analysis[1].



Tetrahedron

[1] See Cook, R.D., *Concepts and Applications of Finite Element Analysis,* 4th ed., Wiley, Hoboken, NJ, 2002, p. 259.

1. Preprocessing, cont.

Mesh generation (discretization)

Global mesh parameters and stats:

Mesh Details

Study name	Static 4 - Torque_Simple_supports (-Default-)
Mesh type	Solid Mesh
Mesher Used	Curvature-based mesh
Jacobian points	16 points
Mesh Control	Defined
Max Element Size	0.583111 in
Min Element Size	0.116622 in
Mesh quality	High
Total nodes	533526
Total elements	353696
Maximum Aspect Ratio	54.979
Percentage of elements with Aspect Ratio < 3	97.5
Percentage of elements with Aspect Ratio > 10	0.0486
% of distorted elements (Jacobian)	0
Time to complete mesh(hh:mm:ss)	00:00:22
Computer name	

Local mesh parameters for stress concentration regions:

- Grooves and keyways:

MeshControl Details

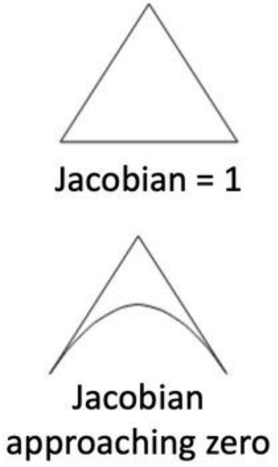
Study name	Static 4 - Torque_Simple_supports (-Default-)
Mesh type	Solid Mesh
Entities	16 face(s)
Units	in
Size	0.05
Ratio	1.5
Identifier	1

- Shoulders (gear shafts to center):

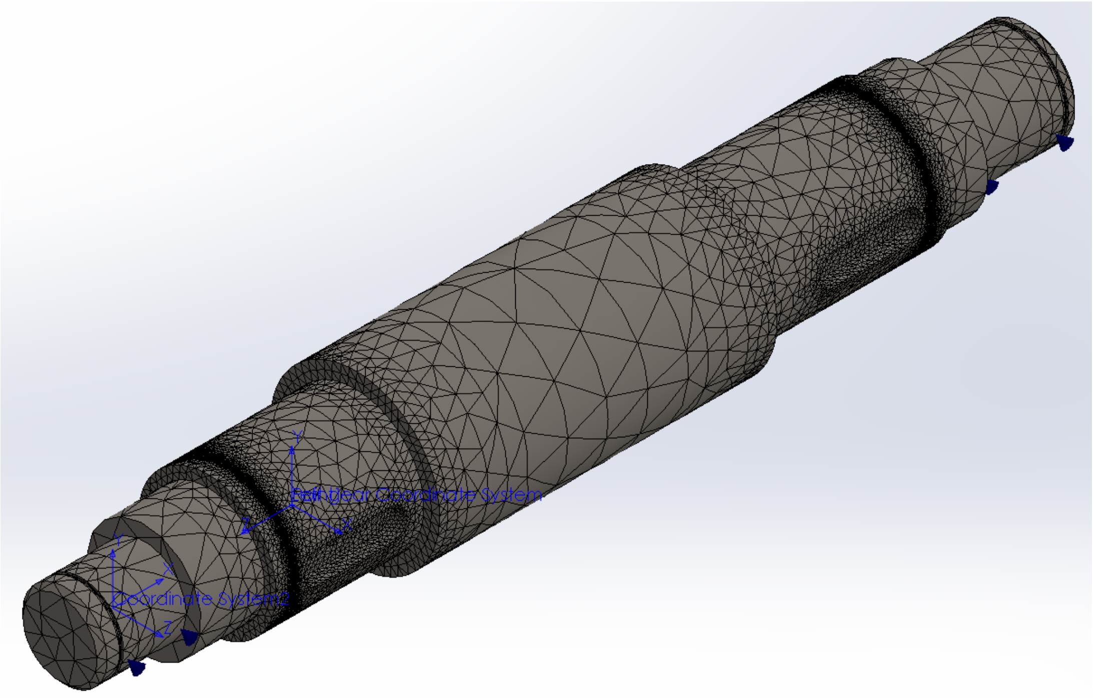
MeshControl Details

Study name	Static 4 - Torque_Simple_supports (-Default-)
Mesh type	Solid Mesh
Entities	8 face(s)
Units	in
Size	0.1
Ratio	1.5
Identifier	2

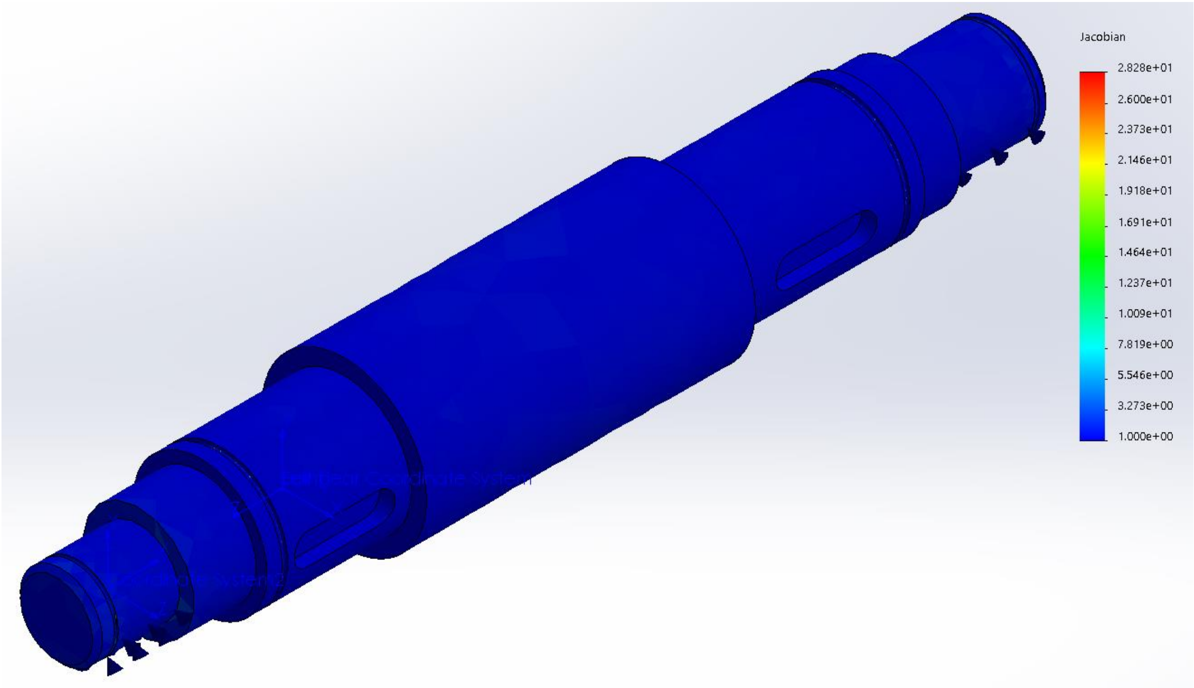
1. Preprocessing, cont.



Mesh (discretized model)



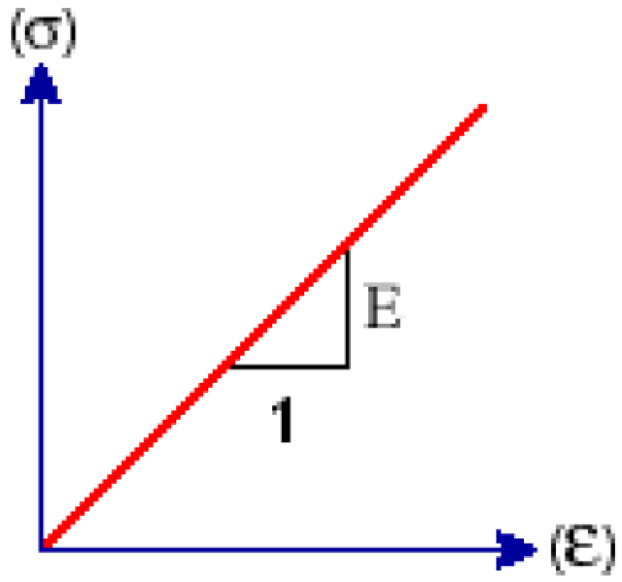
Meshed Model



Mesh Quality Plot (Jacobian)

2. Numerical Analysis

- **Type of Analysis:**
  - Linear elastic isotropic



Engineering stress-strain elastic region

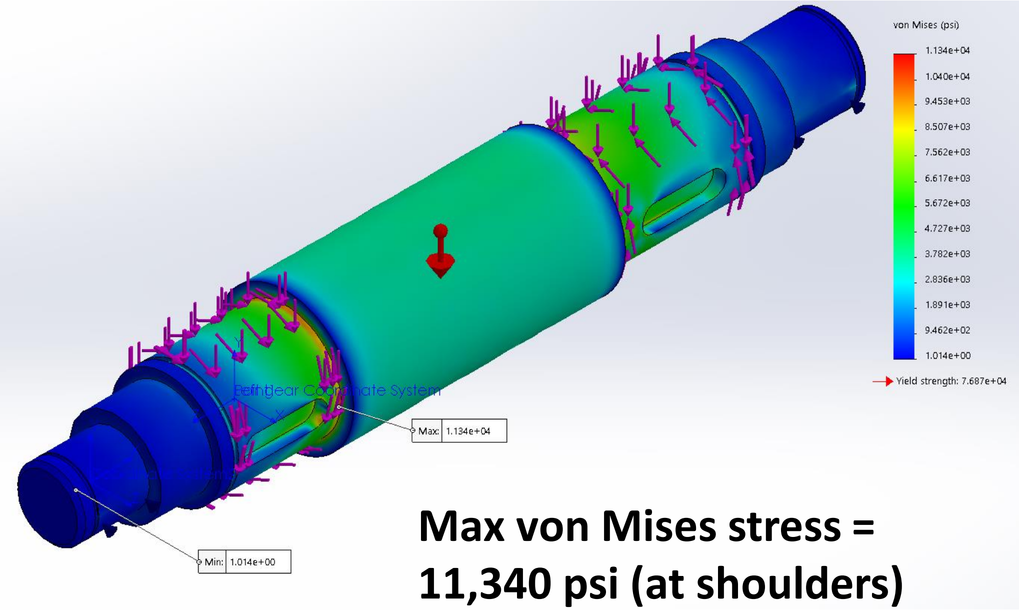
- Linear elasticity
  - Stress-strain relationship of a stress-element of the isotropic case is given by:

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} = \begin{bmatrix} 1/E & -v/E & -v/E & 0 & 0 & 0 \\ -v/E & 1/E & -v/E & 0 & 0 & 0 \\ -v/E & -v/E & 1/E & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix}$$

Stress-strain relationship

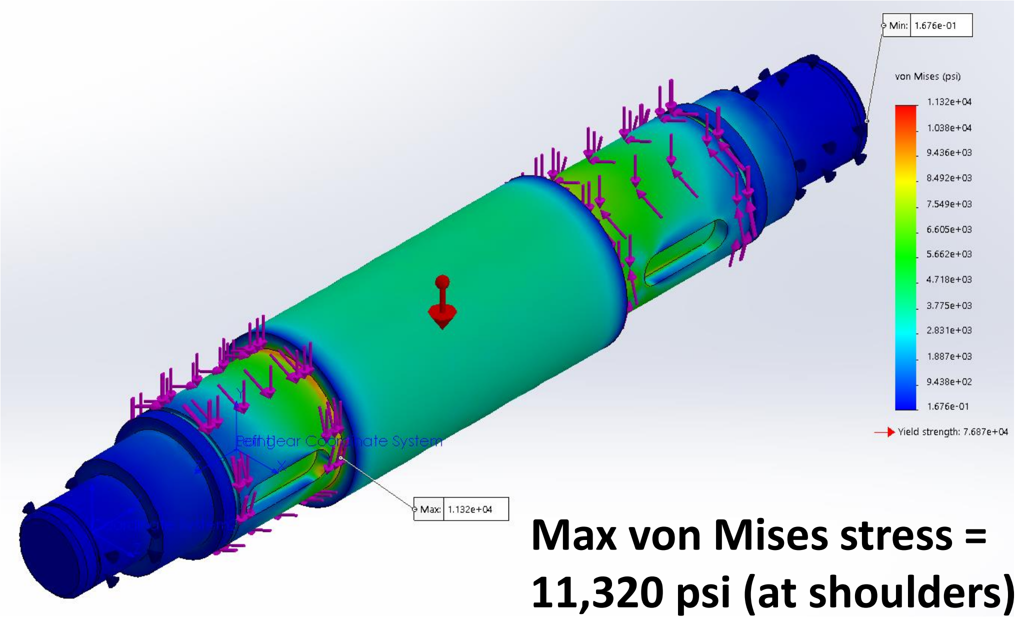
3. Postprocessing

- **Stress plots:** **(AISI 1045 Steel, cold drawn, yield strength = 76,870 psi)**
  - Load case 1: Radial loads and torque with simple supports
  - Load case 1: Radial loads and torque with fixed supports



Max von Mises stress = 11,340 psi (at shoulders)

Stress plot – Simple supports

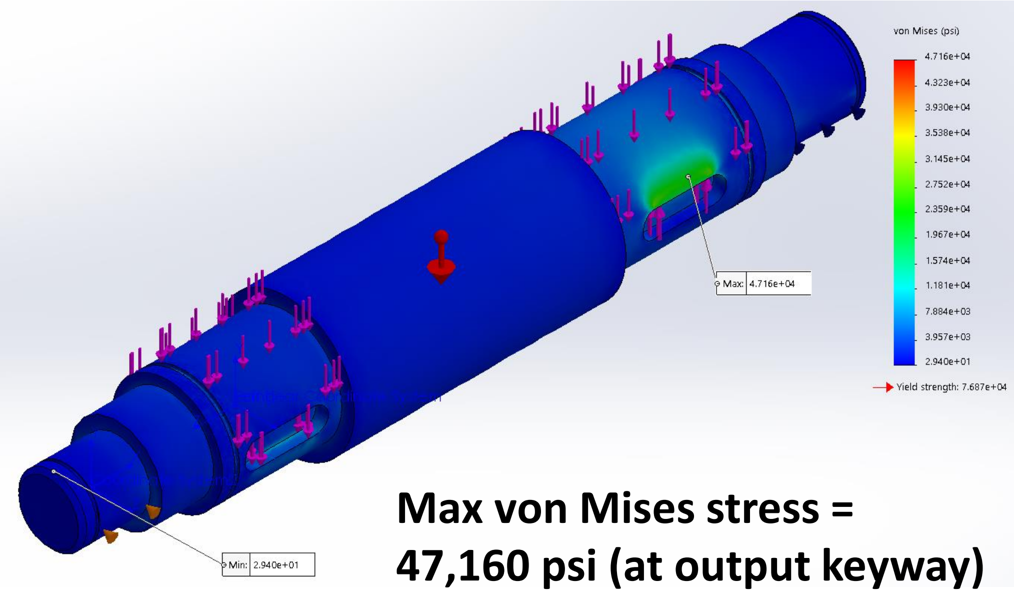


Max von Mises stress = 11,320 psi (at shoulders)

Stress plot – Fixed supports

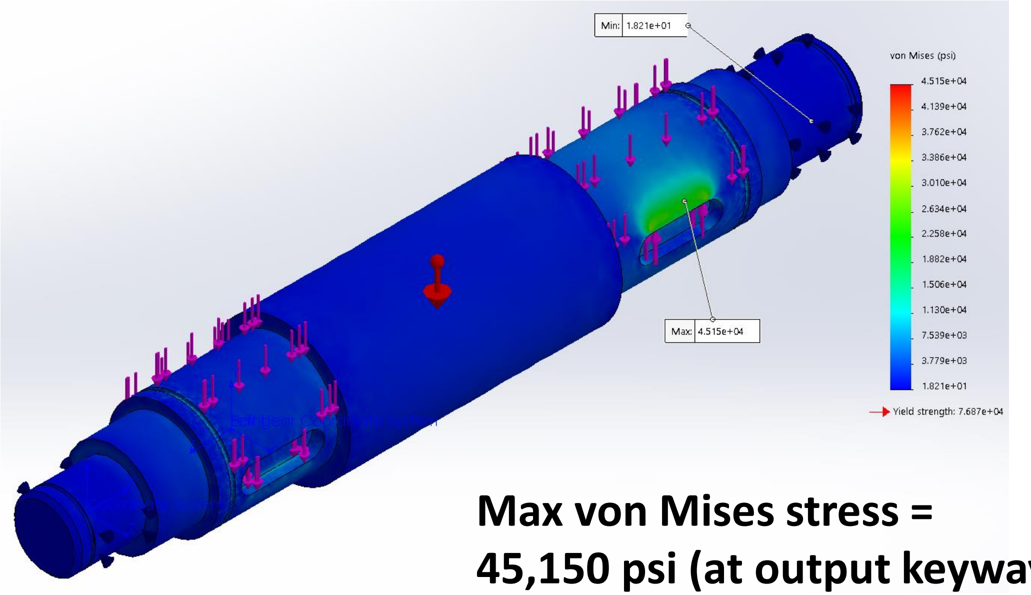
3. Postprocessing, cont.

- **Stress plots:** **(AISI 1045 Steel, cold drawn, yield strength = 76,870 psi)**
  - Load case 2: Radial loads and transmitted force with simple supports
  - Load case 2: Radial loads and transmitted force with fixed supports



Max von Mises stress = 47,160 psi (at output keyway)

Stress plot – Simple supports

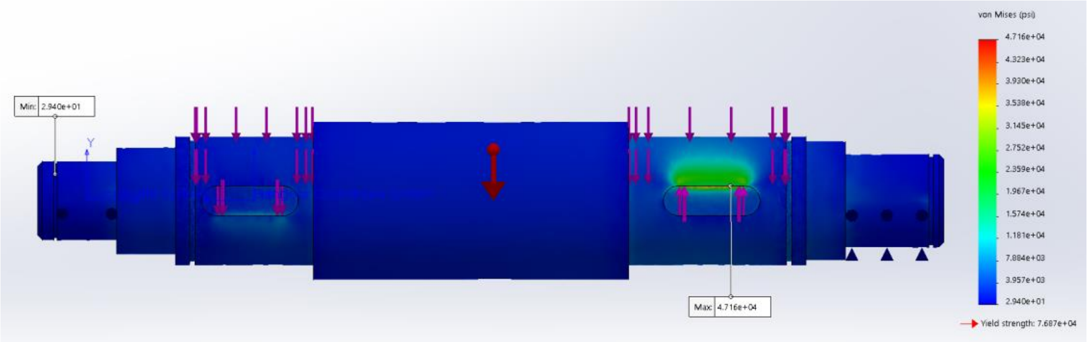


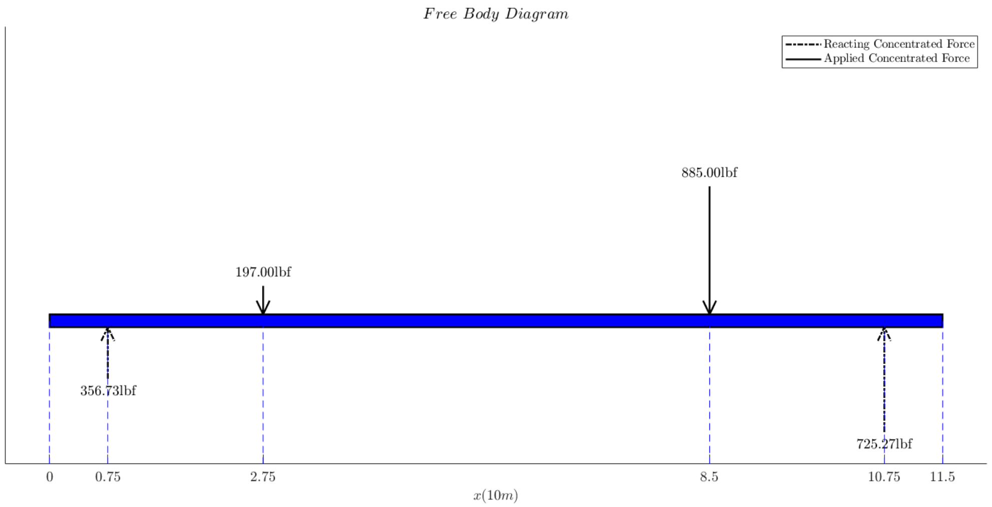
Max von Mises stress = 45,150 psi (at output keyway)

Stress plot – Fixed supports

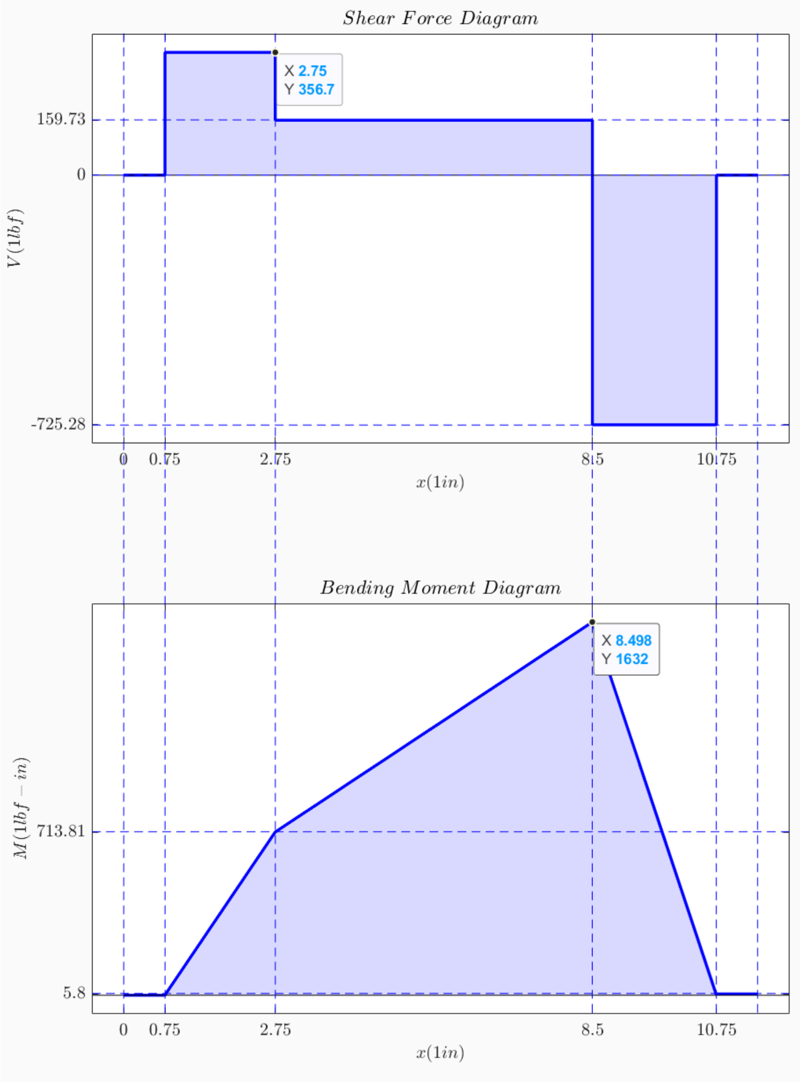
Closed-Form Solutions – Bending Force Analysis

- x-y plane:





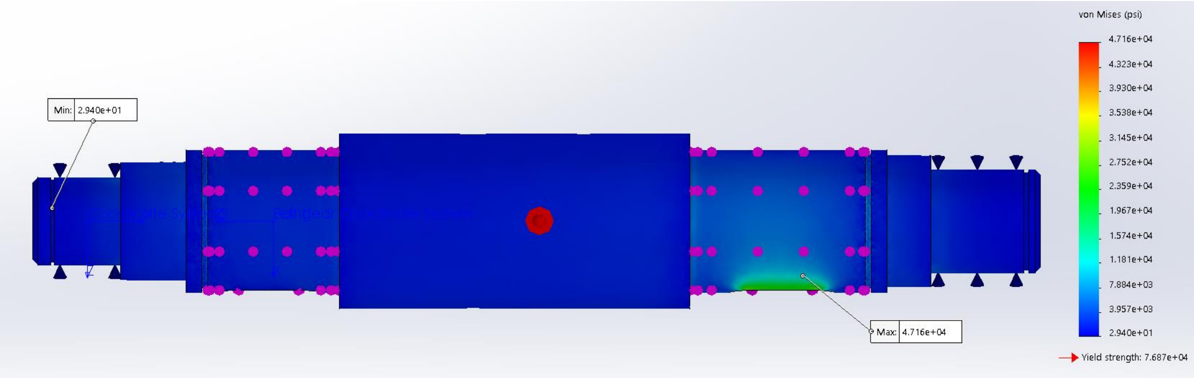
Range	Equations of Shear Force :	Equations of Bending Moment :
0 to 0.75	0	0
0.75 to 2.75	356.725	$356.725x - 266.652$
2.75 to 8.5	159.725	$159.725x + 274.803$
8.5 to 10.75	-725.275	$-725.275x + 7795.975$
10.75 to 11.5	0	5.796

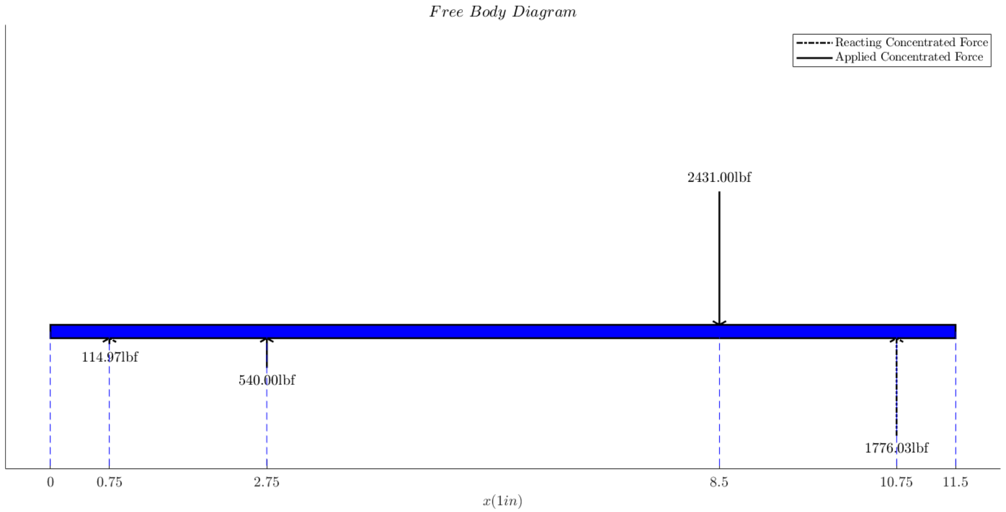


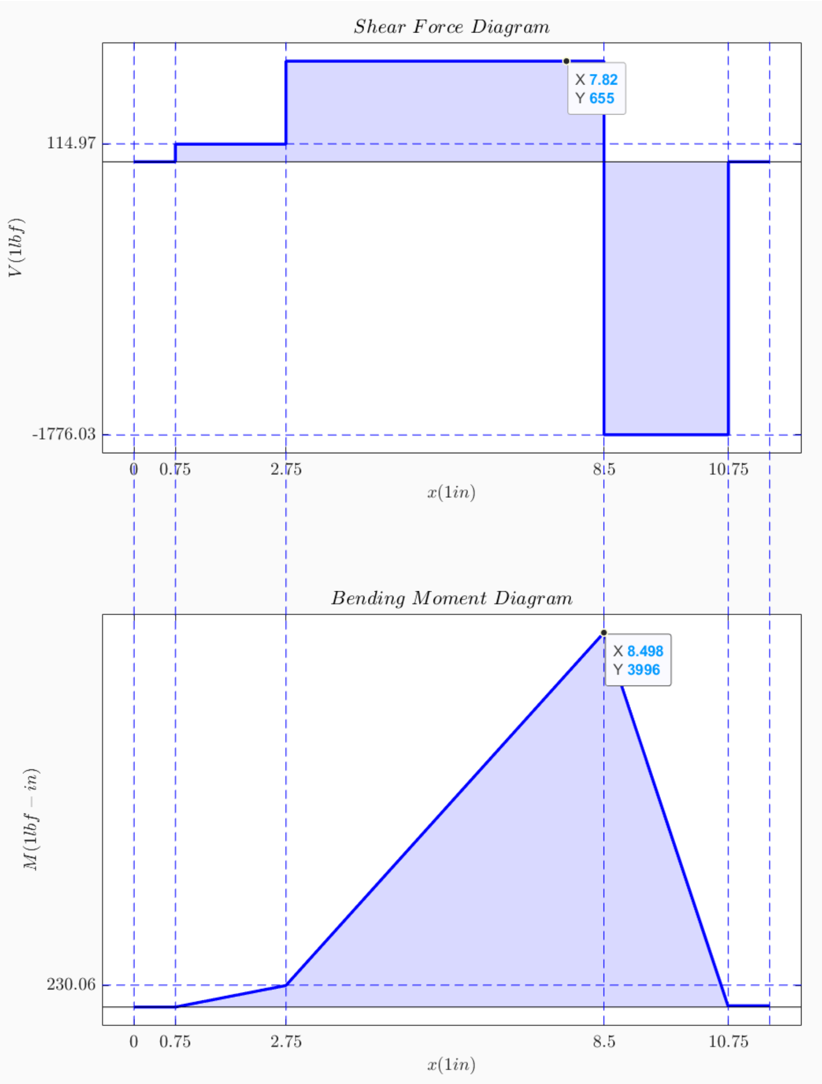
*Note: Transmitted loads are being simulated as normal forces on the keyway side faces (shown here acting along the y-axis); in the closed-form solution, transmitted loads are analyzed as tangential forces (acting along the z-axis) on the shaft's surface.

Closed-Form Solutions – Bending Force Analysis

- x-z plane:



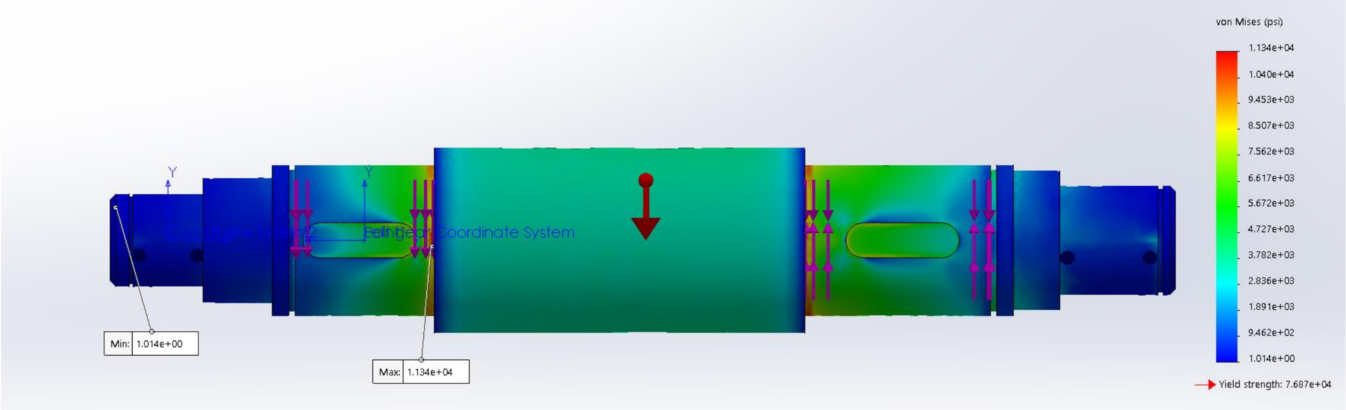


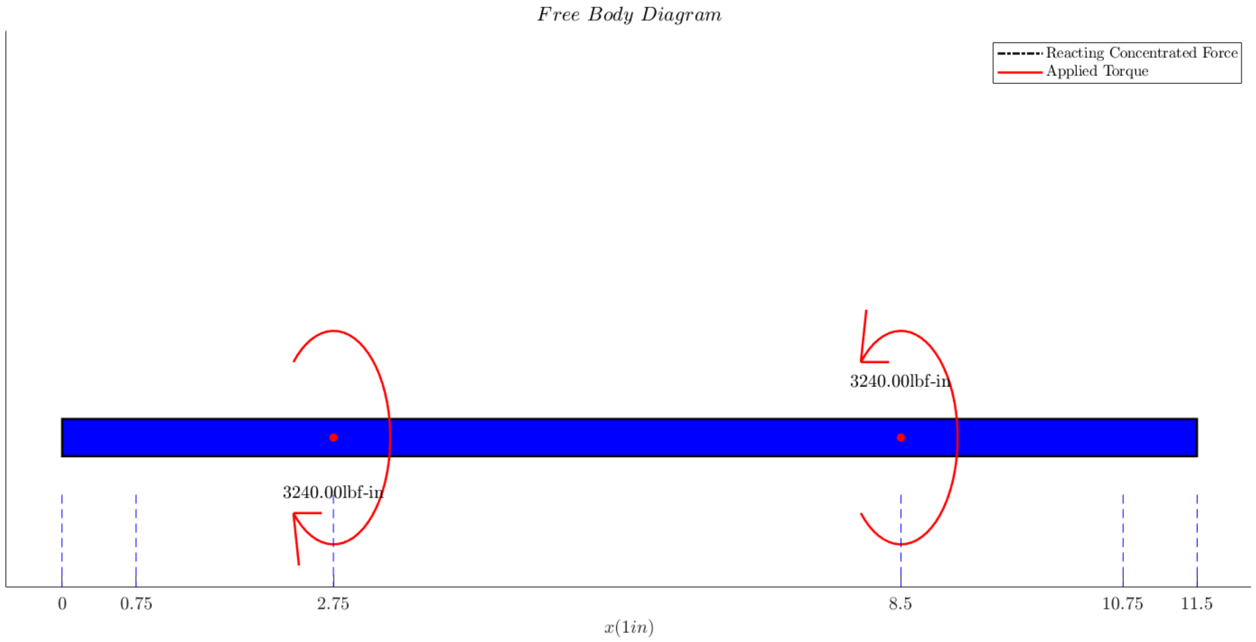


Range	*Equations of Shear Force :*	*Equations of Bending Moment :*
0 *to* 0.75	0	0
0.75 *to* 2.75	114.975	$114.975x - 85.944$
2.75 *to* 8.5	654.975	$654.975x - 1570.134$
8.5 *to* 10.75	−1776.025	$-1776.025x + 19089.72$
10.75 *to* 11.5	0	13.435

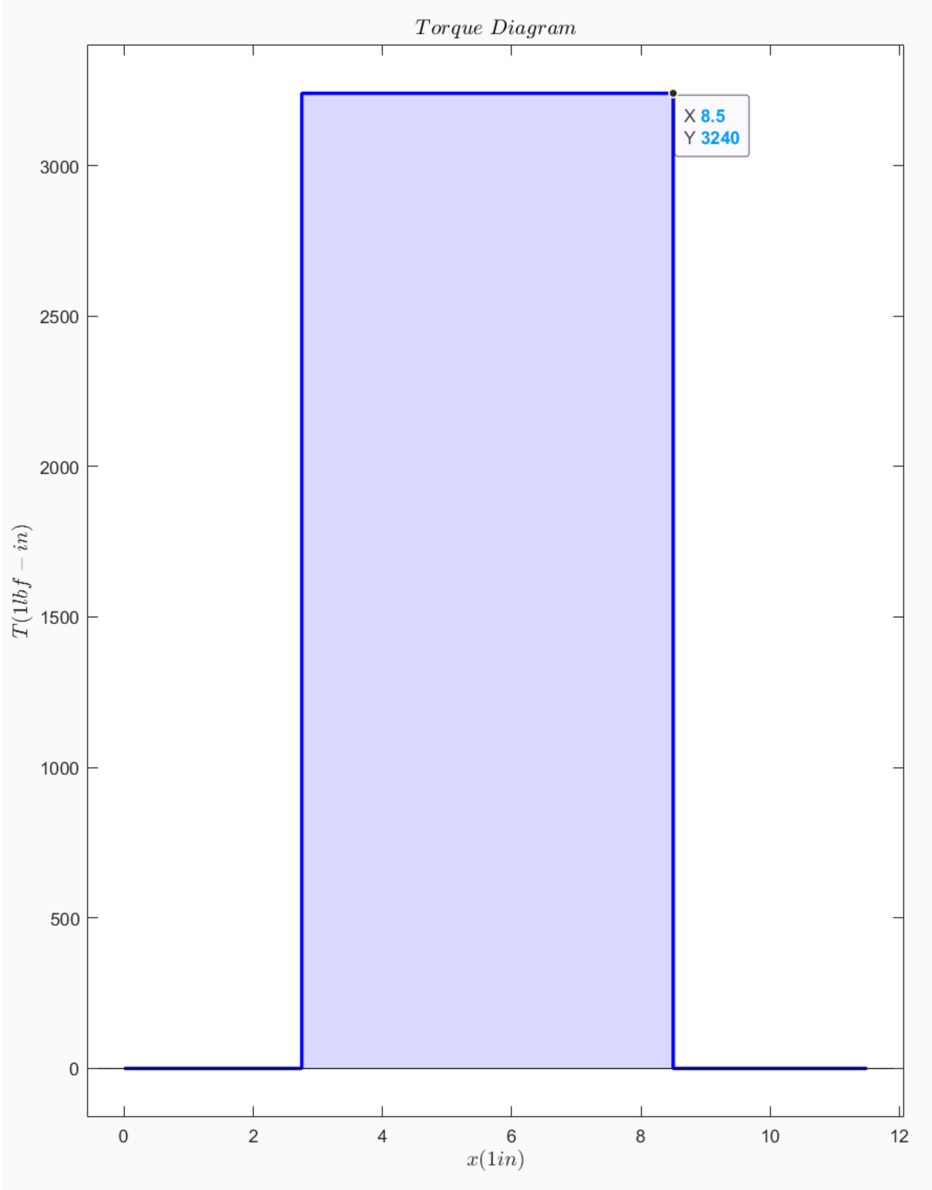
*Note: Transmitted loads are being simulated as normal forces on the keyway side faces (shown here acting along the y-axis); in the closed-form solution, transmitted loads are analyzed as tangential forces (acting along the z-axis) on the shaft's surface.

Closed-Form Solutions – Torsion Force Analysis





Range	*Equations of Shear Force :*	*Equations of Bending Moment :*
0 *to* 0.75	0	0
0.75 *to* 2.75	0	0
2.75 *to* 8.5	0	3240
8.5 *to* 10.75	0	0
10.75 *to* 11.5	0	0

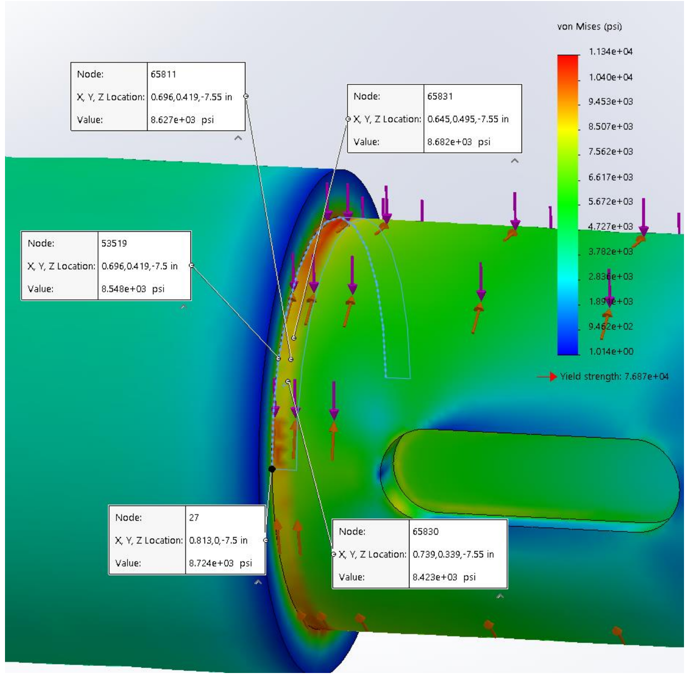


Closed-Form Solutions – Estimated Stress Concentrations Compared to Computational Results

- Von Mises stresses at right shoulder of shaft (gear 4 shaft) due to torsion[1]
  - $\sigma'_m = \left[3\left(\frac{16K_{fs}T_m}{\pi d^3}\right)^2\right]^{1/2} = \frac{\sqrt{3}(16)(1.33)(3240)}{\pi(1.625)^3} = 8{,}859\ psi$
    - Where,
      - $\sigma'_m = von\ Mises\ stress\ due\ to\ midrange\ torque$
      - $K_{fs} = fatigue\ stress\ concentration\ factor\ for\ torsion$
        - From charts of theoretical stress-concentrations[2]
      - $T_m = Torsion$
        - Transmitted load at gear 3 x gear 3 radius

- The computational results agree with the calculated stress concentration.

Computational Results – Load case 1, simple supports:



Avg von Mises stress = 8,601 psi (at right shoulder)

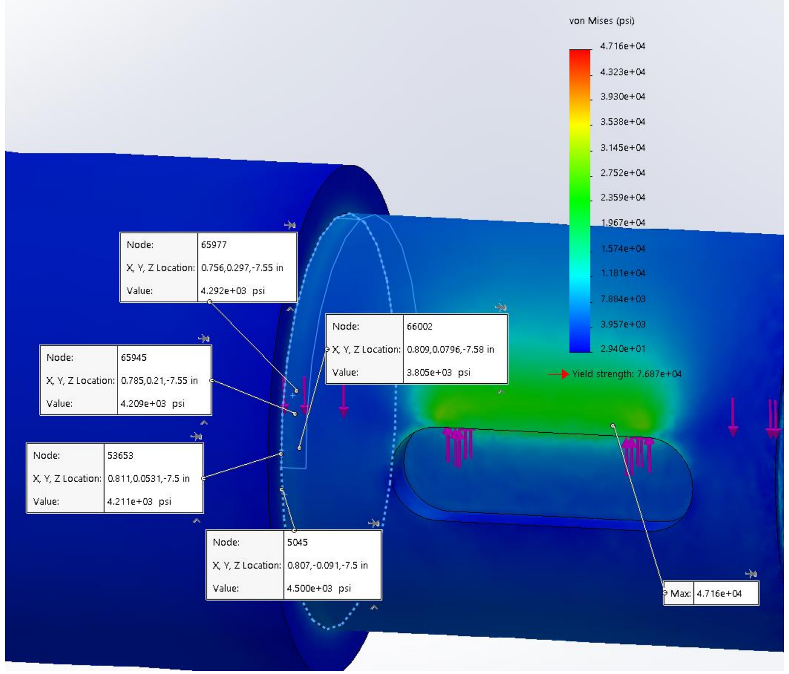
[1]See Budynas, R.G., *Shigley's Mechanical Engineering Design,* 8th ed., McGraw-Hill, New York, NY, 2008, pp. 360 - 367.
[2] See Budynas, R.G., *Shigley's Mechanical Engineering Design,* 8th ed., McGraw-Hill, New York, NY, 2008, pp. 1008.

Closed-Form Solutions – Estimated Stress Concentrations Compared to Computational Results

- Von Mises stresses at right shoulder of shaft (gear 4 shaft) due to bending moment[1]
  - $\sigma_a' = \dfrac{32K_f M_a}{\pi d^3} = \dfrac{32(1.49)(3651)}{\pi(1.625)^3} = 12{,}910\ psi$
    - Where,
      - $\sigma_a' = von\ Mises\ stress\ due\ to\ alternating\ bending\ moment$
      - $K_f = fatigue\ stress\ concentration\ factor\ for\ bending$
        - From charts of theoretical stress-concentrations[2]
      - $M_a = \ total\ moment\ at\ right\ shoulder$
        - Calculated by combining orthogonal planes as vectors

- The computational results show much more <u>conservative</u> stress as compared to the estimated stress concentration, meaning stresses from the simulation are higher than analytical solutions.

Computational Results – Load case 2, simple supports:



Avg von Mises stress = 42,030 psi (at right shoulder)

[1]See Budynas, R.G., *Shigley's Mechanical Engineering Design,* 8th ed., McGraw-Hill, New York, NY, 2008, pp. 360 - 367.
[2] See Budynas, R.G., *Shigley's Mechanical Engineering Design,* 8th ed., McGraw-Hill, New York, NY, 2008, pp. 1008.

Conclusion

- Load case 1:
  - Computational results agree with closed-form solutions.
- Load case 2:
  - Computational results are higher than those estimated with closed-form solutions, indicating analysis is conservative.
- Because the von Mises stresses, both the computational and analytical, are *less* than the yield strength of the material, it is expected that the component will withstand operational loads.
  - Factor of safety[1]: $n = \frac{S_y}{\sigma'} = \frac{76{,}870}{42{,}030} = 1.83$

[1]See Distortion-Energy Theory for Ductile Materials in Budynas, R.G., *Shigley's Mechanical Engineering Design,* 8th ed., McGraw-Hill, New York, NY, 2008, pp. 213 - 217.